

Characterization of f-derivation of BF-Algebra

Sardar Muhammad¹, Muhammad Idrees²

¹University of Balochistan, Quetta-Pakistan. sardar_77@yahoo.com

²Government Degree College, Quetta-Pakistan. idrees.math@gmail.com

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ABSTRACT

In the manuscript we lay down the foundation of concept of compositions of derivation of BF-algebra, also lay down the structure of f-derivation of BF-algebras and investigated some of the properties and derive the result which is related to BF-algebra. We prove in general the propositions, and properties is proved.

Keywords: BF-algebra, derivations, composition of derivations, f-derivations:

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INTRODUCTION

Background

This is preminarily section which aim is to introduce some of the results and properties which is useful for derivation of oncoming section, First we will present the definition of BF-algebra and the some properties, which is given below,

Definition:1.1 [10] A BF-algebra is a non-empty set A with a consonant 0 and a binary operation $*$ satisfying the following axioms:

$$(F_a) a * a = 0,$$

$$(F_b) a * 0 = a,$$

$$(F_c) 0 * (a * b) = (b * a),$$

For all $a, b \in A$.

Example:1.2: Let R be the set of real numbers and let $A = (R; *, 0)$ be the algebra with the operation $*$ defined by

$$a * b = \begin{cases} a & b = 0 \\ b & a = 0 \\ 0 & \text{otherwise} \end{cases}$$

Then Z is a BF-algebra.

Definition:1.3: [1] A non-empty subset S of a BF-algebra A is called a sub algebra of A if $a * b \in S$ for any $a, b \in S$.

Proposition:1.4 Let $(A, *)$ be a BF -algebra. Then the following results hold in Algebra

(F₁) Z is a BG-algebra;

(F₂) For all $a, b \in Z$, $a * b = 0$ implies $a = Z$;

(F₃) The right cancellation law holds in Z . i.e.,

If $a * b = c * b$, then $a = c$ for all $a, b, c \in Z$;

(F₄) The left cancellation law holds in Z . i.e.

If $b * a = b * c$, then $a = c$ for all $a, b, c \in Z$

(F₅) $0 * (0 * a) = a$ for all $a \in Z$;

(F₆) If $0 * a = 0 * b$, then $a = b$ for all $a, b \in Z$;

(F₇) If $a * b = 0$, then $b * a = 0$ for all $a, b \in Z$.

(F₈) $(a * z) * (b * c) = a * b$ for all $a, b, c \in Z$

(F₉) $(a * (0 * a)) * a = a$

MAIN RESULTS

Definition 2.1:- Let $(Z, *, 0)$ is an associative BF-algebra and $\eta : Z \rightarrow Z$ is a self-map then η called (*Left, Right*)-derivation on Z if $\eta (a * b) = (\eta (a) * b) \wedge (a * \eta (b))$.

Definition 2.2:- Let $(Z, *, 0)$ is an associative BF-algebra and $\eta : Z \rightarrow Z$ is a self-map then η is called (*Right, left*)-derivation on Z if $\eta (a * b) = (a * \eta (b)) \wedge (\eta (a) * b)$.

Definition 2.3:- If η is both (Left-Right)-derivation and (Right-Left)-derivation on Z then η is called derivation on Z .

Definition 2.4:- A self-map $\eta : Z \rightarrow Z$ on associative BF-algebra Z is called regular if $\eta (0) = 0$.

Example 2.5:- Let the set $Z = \{0, a, b, c\}$ defined by the following table.

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Is a BF-algebra and a map, $\eta : Z \rightarrow Z$ defined by

$\eta (0) = c$ $\eta (a) = b$ $\eta (b) = a$ and $\eta (c) = 0$ is both (*Left – Right*)-derivation and (Right-left)-derivation on Z and thus a derivation of Z .

Definition: 2.6: Let Z be a BF-algebra and, be two self-maps of Z . We define $\eta_1 \circ \eta_2 : Z \rightarrow Z$ such that $\eta_1 \circ \eta_2 (a) = \eta_1 (\eta_2 (a))$ for all, $a, b \in Z$

Proposition 2.7: Let Z be a BF-algebra. Let η_1 and η_2 be two left-right derivations of Z . Then $(\eta_1 \circ \eta_2)$ is also a left right derivation of Z .

Proof: suppose that

$$\begin{aligned}
 (\eta_1 \circ \eta_2)(a * b) &= \eta_1(\eta_2(a * b)) \\
 &= \eta_1(\eta_2(a) * b) \wedge (a * \eta_2(b)) \\
 &= (a * \eta_2(b) * (a * \eta_2(b) * (\eta_2(a) * b))) \\
 &= \eta_1(\eta_2(a) * b) \\
 &= (a * \eta_1(\eta_2(b))) * [(a * \eta_1(\eta_2(b)) * (\eta_1(\eta_2(a) * b))] \\
 &= (a * \eta_1 \circ \eta_2)(b) * [(a * \eta_1 \circ \eta_2)(b) * ((\eta_1 \circ \eta_2)(a) * b)] \\
 &= ((\eta_1 \circ \eta_2)(a) * b) \wedge (a * (\eta_1 \circ \eta_2)(b))
 \end{aligned}$$

Hence $\eta_1 \circ \eta_2$ is Left- right derivation of BF--algebra Z.

Similarly we can prove the composition of two right-left derivation is also the right-left derivation of BF- algebra Z.

Proposition: 2.8 Let Z be a BF-algebra. Let η_1 and η_2 are (right, left) –derivations of Z. Then $\eta_1 \circ \eta_2$ is also a (Right, Left)-derivation of BF-algebra Z.

Proof:

$$\begin{aligned}
 (\eta_1 \circ \eta_2)(a * b) &= \eta_1(\eta_2(a * b)) \\
 &= \eta_1[(a * \eta_2(b)) \wedge (\eta_2(a) * b)] \\
 &= \eta_1[(\eta_2(a) * b) * ((\eta_2(a) * b) * a * \eta_2(b))] \\
 &= \eta_1(a * \eta_2(b)) \text{ Because PU-algebra is associative} \\
 &= (a * \eta_1(\eta_2(b))) \wedge (\eta_1(a) * \eta_2(b)) \\
 &= (\eta_1(\eta_2(a) * b) * [\eta_1(\eta_2(a) * b) * (a * \eta_1(\eta_2(b)))] \\
 &= ((\eta_1 \circ \eta_2)(a) * b) * [((\eta_1 \circ \eta_2)(a) * b) * (a * (\eta_1 \circ \eta_2)(b))] \\
 &= (a * (\eta_1 \circ \eta_2)(b)) \wedge ((\eta_1 \circ \eta_2)(a) * b)
 \end{aligned}$$

Therefore the composition of two (Right-left) mapping in BF-algebra is right mapping.

Proposition.2.9: Let $(Z, *, 0)$ be a BF-algebra and η_1 and η_2 are two derivations of Z. Then $\eta_1 \circ \eta_2$ is also a derivation of BF-algebra Z,

Proof:

Straightforward.

f-derivation of BF-algebra

In this section we will present some main results which is related to f-derivation of BF-algebra, and derive some results of f-derivation BF-algebra, the discussion are given below,

Definition 3.1:- Let $(Z, *, 0)$ is a associative BF-algebra and $\eta_f : Z \rightarrow Z$ is a self-map then η_f

called (**Left, Right**)- f derivation on Z if $\eta_f(a * b) = (\eta_f(a) * f(b)) \wedge (f(a) * \eta_f(b))$.

Definition 3.2:- Let $(Z, *, 0)$ is an associative BF-algebra and $\eta_f : Z \rightarrow Z$ is a self-map then η_f

is called (**Right, left**)-derivation on Z if $\eta_f(a * b) = (f(a) * \eta_f(b)) \wedge (\eta_f(a) * f(b))$.

Definition 3.3:- If η_f is both (**Left, Right**)-derivation and (**Right, left**)-derivation on Z then η_f is called f-derivation on Z .

Definition 3.4: A self-map $\eta_f : Z \rightarrow Z$ on BF-algebra Z is called regular if $\eta_f(0) = 0$.

Example 3.5:- Let the set $Z = \{0, a, b, c\}$ defined by the following table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Is a BF-algebra and a map, $\eta_f : Z \rightarrow Z$ defined by

$\eta_f(0) = 3$ $\eta_f(1) = 2$ $\eta_f(2) = 1$ and $\eta_f(3) = 0$ is both (*Left – Right*)-derivation and (Right-left)-f derivation on $\mathbb{Z}$ and thus f derivation of $\mathbb{Z}$.

Proposition 3.6:- Let η_f be a (*Left, Right*)-f-derivation of an associative BF-algebra $\mathbb{Z}$ then

(F₉): $\eta_f(0) = \eta_f(a) * a, \forall a \in \mathbb{Z}$.

(F₁₀): η_f is one-one map.

(F₁₁): If η_f is a regular map then it is identity.

(F₁₂): If there exists an element $a \in \mathbb{Z}$ such that $\eta_f(a) = a$ then the map η_f is identity.

(F₁₃): If $\eta_f(b) * f(a) = 0$ or $f(a) * \eta_f(b) = 0$ then $\eta_f(b) = f(a), \forall a, b \in \mathbb{Z}$ i.e. η_f is constant.

Proof (F₉): $\eta_f(0) = \eta_f(a * a), \because$ by (F_a)

$$\begin{aligned} &= (\eta_f(a) * f(a)) \wedge (f(a) * \eta_f(a)) \\ &= (f(a) * \eta_f(a)) * [(f(a) * \eta_f(a)) * (\eta_f(a) * f(a))] \\ &= (f(a) * \eta_f(a)) * [(0 * (\eta_f(a) * f(a)) * (\eta_f(a) * f(a))] \\ &= (\eta_f(a) * f(a)), \quad \because \text{by (F}_9\text{)} \end{aligned}$$

Proof (F₁₀): Let $a, b \in \mathbb{Z}$ such that $\eta_f(a) = \eta_f(b) \dots\dots\dots$ (i)

From (F₉) we have $\eta_f(0) = \eta_f(a) * f(a) \quad \forall a \in \mathbb{Z} \dots\dots\dots$ (ii)

Also from (F₉) we have $\eta_f(0) = \eta_f(b) * f(b) \quad \forall b \in \mathbb{Z} \dots\dots\dots$ (iii)

From (ii) and (iii) we get $\eta_f(a) * f(a) = \eta_f(b) * f(b) \dots\dots\dots$ (iv)

Using the result of equation (i) in equation (iv) we get

$$\eta_f(a) * f(a) = \eta_f(a) * f(b) \dots\dots\dots (v)$$

By (F₄) left cancellation law holds in $\mathbb{Z}$ therefore from (v) we get $a = b$.

Hence η_f is one to one.

Proof (F₁₁): Let η_f is regular then we have $\eta_f(0) = 0$ (i)

From (F_a) we have $\eta_f(0) = \eta_f(a) * f(a) \quad \forall a \in \mathbb{Z}$ (ii)

From (i) and (ii) we get $\eta_f(a) * f(a) = 0 \quad \forall a \in \mathbb{Z}$ (iii)

Now by using (F_a) in the right hand side of equation (iii) then (iii) becomes

$$\eta_f(a) * f(a) = f(a) * f(a) \quad \forall a \in \mathbb{Z} \text{ (iv),}$$

By (F₃) right cancellation law holds in $\mathbb{Z}$ therefore (iv) becomes $\eta_f(a) = f(a) \quad \forall a \in \mathbb{Z}$.

Hence η_f is the identity map.

Proof (F₁₂): Let $\eta_f(a) = f(a)$ (i)

Now by proposition (P₇) equation (i) is equivalent to $\eta_f(a) * f(a) = f(a) * f(a)$

$$\Rightarrow \eta_f(a) * f(a) = 0 \text{ (ii), } \because \text{ by (F}_1\text{)}$$

From (F₉) we have $\eta_f(0) = \eta_f(a) * f(a) \quad \forall a \in \mathbb{Z}$ (iii)

So now using the result of equation (iii) in the left hand side of equation (ii) we get

$$\eta_f(0) = 0 \Rightarrow \eta_f \text{ the identity map.}$$

Proof (F₁₃): Let $\eta_f(b) * f(a) = 0$ (i),

By proposition (F_a) equation (i) becomes $\eta_f(b) * f(a) = f(a) * f(a)$ (ii), by (F₃) right cancellation law holds in $\mathbb{Z}$ therefore (ii) becomes $\eta_f(b) = f(a)$.

Similarly for $f(a) * \eta_f(b) = 0$ (iii), by (F_a) equation (iii) becomes $f(a) * \eta_f(b) = f(a) * f(a)$ (iv), by (F₄) left cancellation law holds in $\mathbb{Z}$ therefore (iv) becomes $\eta_f(b) = f(a)$. $\Rightarrow$ η_f is constant.

So now using (iii) in (ii) we get $\eta_f(0) = 0 \Rightarrow \eta_f$ is regular which by (P₁₆) η_f is the identity map.

Proof (P₁₄):

Let $\eta_f(b) * f(a) = 0 \dots (i)$,

By proposition (P₃) the right hand side of equation (i) becomes

$$\eta_f(b) * f(a) = f(a) * f(a) \dots (ii)$$

By (P₈) right cancellation law holds in Z therefore (ii) becomes $\eta_f(b) = f(a)$

Similarly $f(a) * \eta_f(b) = 0 \dots (iii)$

By (P₃) equation (iii) becomes

$$f(a) * \eta_f(b) = f(a) * f(a) \dots (iv)$$

By (P₈) left cancellation law holds in Z therefore (iv) becomes $\eta_f(b) = f(a)$.

Theorem 3.7: Let Z is a BF-algebra

(F_d): If η_f is (*left, Right*)- f -regular derivation of Z then $\eta_f(a) = f(a) \wedge \eta_f(a) \quad \forall a \in Z$.

(F_e): If η_f is (*Right, left*)- f -regular derivation of Z then $\eta_f(a) = \eta_f(a) \wedge f(a) \quad \forall a \in Z$.

Proof (F_d): Since η_f is regular therefore we have $\eta_f(0) = 0 \dots (i)$

Now consider for some $a \in Z$ we have $\eta_f(a) = \eta_f(a * 0)$, $\because$ by (F₂)

$$= (\eta_f(a) * f(0)) \wedge (f(a) * \eta_f(0)),$$

$$= ((\eta_f(a) * 0) \wedge (f(a) * 0)), \quad \because \text{by using (F}_2\text{)}$$

$$= \eta_f(a) \wedge f(a),$$

Proof (F_e): Since η_f is regular therefore we have $\eta_f(0) = 0 \dots (i)$

Now consider for some $a \in Z$, $\eta_f(a) = \eta_f(a * 0)$, $\because$ by (F₂)

$$= (f(a) * \eta_f(0)) \wedge (\eta_f(a) * f(0)) \quad \because \text{by definition 3.1}$$

$$= (f(a) * 0) \wedge (\eta_f(a) * 0)$$

$$= f(a) \wedge \eta_f(a) \quad \because \text{by (F}_2\text{)}$$

Theorem 3.8: Let η_f is a self-map of a BF-Algebra Z then

$$(f(a) * (f(a) * \eta_f(a))) * f(a) = (\eta_f(a) * (\eta_f(a) * f(a))) * f(a).$$

Proof: Since by (theorem 3.7 (F_e)) we have,

$$\eta_f(a) = \eta_f(a) \wedge f(a) = f(a) * (f(a) * \eta_f(a)) \dots (i)$$

Equation (i) is equivalent to

$$\eta_f(a) * f(a) = (f(a) * (f(a) * \eta_f(a))) * f(a) \dots (ii).$$

On the other hand from (theorem 3.7 (F_d)) we have

$$\eta_f(a) = f(a) \wedge \eta_f(a) = \eta_f(a) * (\eta_f(a) * a) \dots (iii).$$

Similarly equation (iii) is equivalent to

$$\eta_f(a) * f(a) = (\eta_f(a) * (\eta_f(a) * f(a))) * f(a) \dots (iv).$$

From (ii) and (iv) we get

$$(f(a) * (f(a) * \eta_f(a))) * a = (\eta_f(a) * (\eta_f(a) * f(a))) * f(a).$$

Objective

Following are the objectives of our research

To analyses the mathematical structure of BF-algebra. We will derive the main results which is concern with the theory of derivations. Further we will investigate the derivation of f-derivation and their composition. We will also analysis the algebraic structure of composition of f-derivation and derivate very crucial results related to the theory of BF-algebra. We will formulate the effectiveness of derivation of BF-algebra with other algebraic results. Lastly the derivation results will be compare with other algebraic results.

LITERATURE REVIEW

(Andrzes Walendziak, 2013) introduce the structure BF-algebra obtained some of the properties and derive some essential mathematical results which paved the way for other algebraic results as well to achieve same results. Also BCK-algebras and BCI-algebras are the important categories of abstract algebras and these algebras were initiated by the eminent mathematicians (Imai & Iseki, 1966). BCI-Algebra has a vast space than that of BCK-Algebra therefore BCK-Algebra is considered as its proper subclass. BCH-Algebra is another class of an abstract algebra which was introduced by (Hu & Li, 1983). It is a huge class of logical algebras due to which BCI-algebra occurs as its proper subclass. The concept of d-algebras were given by (Neggers & Kim, 1999). The d-algebra is observed as a generalization of BCK-Algebra. There is another type of abstract algebras termed as TM-algebra. This algebra is a generalization of BCH / BCI / BCK-Algebras while the foundation of such an algebra was laid down by (Megalai & Tamilarasi, 2010). PU-algebra is a new structure which is considered as a dual for TM-algebra and was launched by (Mostafa. et al., 2015). The concept of derivation in BCI-algebra was inserted by (Jun & Xin, 2004) as a result of motivation by the concept of derivation on rings and near rings. In the field of d-algebras the concept of derivation was laid down by the researchers (Chandramouleeswaran & Ganeshkumar, 2011). The idea of derivations in TM-algebra was established by (Chandramouleeswaran & Ganeshkumar, Derivations on TM-algebras , 2012) . In this manuscript we embedded the concept of derivations in associative PU-Algebra and investigated some of its interesting results.(P. Ganesan et al) laid down the foundation of algebraic structural of composition of derivation of PU-algebra and derive some of the properties.

METHODOLOGY

The research work will be carried out in department of university library and in the department mathematics in Government degree college Quetta. The research work is based on the mathematically structure of derivation of BF-algebra.

In the research paper which is concern with derivation of other algebraic structure which is published in literature is helpful.

The departmental computer lab and computer lab of Degree College for down loading research articles is used for the for derivation of results.

CONCLUSION

In the manuscript we have investigate some of essential properties of f-derivation BF-algebra and derive the theoretical results which is related to BF-algebra, we hope the manuscript will have paved the way to apply in derivation of other algebraic structure applying the same properties and results.

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BIOGRAPHY

Sardar Muhammad

sardar_77@yahoo.com

Department of Mathematics, University of Balochistan.

Muhammad Idrees

idrees.math@gmail.com

Government Degree College, Department of Mathematics.